

B-Math-III Final Exam ; Differential Equations.

Time : 2.00 hrs; Max Mark: 50 ; 9 May 2022

1. A semi-infinite strip $S := \{(x, y) : 0 \leq x \leq l, 0 \leq y < \infty\}$ has a temperature distribution $u(t, x, y)$ satisfying $u(t, 0, y) = u(t, l, y) = 0, t \geq 0, 0 < y < \infty$ and $u(t, x, 0) = f(x), t \geq 0, 0 < x < l$. Find the steady state temperature distribution in S . (15)

2. Define for each $p \in \mathbb{R}$,

$$J_p(x) := \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{x}{2}\right)^{2n+p}}{n!(p+n)!}.$$

a) For each $p \in \mathbb{R}$ show that the series in the RHS is well defined for every $x \in \mathbb{R}$.

b) For $p \geq 0$ show that $J_p(x)$ satisfies Bessel's equation for every x . (11 + 4)

3. Find the general solution of Euler's equation

$$x^2 y'' + pxy' + qy = 0.$$

(15)

4. Find the unique solution of the PDE

$$uu_x + u_y = 1$$

with the initial curve $\Gamma_0 := \{(x, y) : x = s, y = s, 0 \leq s \leq 1\}$ and initial value $u = \frac{1}{2}s$ on Γ_0 . (15)